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MEASUREMENTS ON CURRENT SHEETS IN PLASMAS*

by

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ABSTRACT

The acceleration of a gas undergoing ionization in a crossed electric-magnetic field leads to the formation of current sheets similar to those observed in coaxial plasma guns. The dynamics of such sheets is of great interest to workers in plasma physics in view of their application to controlled thermonuclear fusion experiments and to space propulsion. In the present paper a detailed mapping of the magnetic field in the vicinity of a current sheet is described. Conventional small search coils are used for the detection of this field. From the configuration of this field an attempt is made to understand the processes taking place in the sheet. All of the experiments described in the paper are performed on a plasma coaxial gun of small aspect ratio.

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INTRODUCTION

Current sheets in plasmas occur whenever an abrupt spatial change in the value of the magnetic field takes place inside the body of a plasma. In view of their importance to several applications to investigations on controlled thermonuclear fusion reactions and to space propulsion the dynamics of these sheets have been examined in details by many workers in the field[1,2,3].

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*Work supported by the National Aeronautics and Space Administration and by the Cambridge Air Force Research Center.

1 L. C. Burkhardt and R. H. Lovberg, Phys. Fluids 5, 341 (1962).

2 R. C. Mjolsness, F. L. Ribe and W. B. Riesenfeld, Phys. Fluids, 4, 730 (1961).

3 G. Schmidt, Phys. Fluids, 5, 995 (1962).

A search of the technical literature reveals that there are many investigations on the theoretical study of current sheets of infinite extent[4,5,6]. In an experimental arrangement, however, the finite size of the apparatus will limit the size of the current sheet. The important difference between the infinitely large sheet and the bounded sheet is the appearance of crossed electric field which may appreciably alter the physics of the situation. The paper presented here, considers the case of a current sheet produced by an electric discharge in a gas confined by two infinitely wide parallel electrodes. (Fig. 1) This configuration is not unrealistic since the effect of the width of the electrodes can be obviated in an arrangement which may use coaxial electrodes for instance. The sheet under considerations would then take place in the annular spacing.

The reason the discussion given here is carried on a planar rather than a cylindrical geometry is to eliminate the dependence of the motion on the radial coordinate. Thus allowing the concentration of the discussion on the essential physical phenomena taking place inside the current sheet.

It has been observed and the experimental results of Burkhardt and Lovberg (1) show this very clearly, that highly planar current sheets can be produced in a plasma. Furthermore these sheets appear to propagate with constant velocities. It has been speculated that the reason for the propagation of these sheets as plane sheets even in a coaxial configuration can be ascribed to an ionization front moving in the gas. The velocity of this front

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[4] M. Rosenbluth, R. Garwin and A. Rosenbluth, A.E.C. Rept. LA-1850 (1954).

[5] J. H. Adlam and J. E. Allen, Phil. Mag. 3, 448 (1958).

[6] K. W. Morton, New York University Rept. NYO-9763 (1961).

is determined by the energetics of the situation in which the energy input from the source is used to accelerate and heat the electrons. These in turn continuously ionize the neutral atoms with which they collide. Since the electron-neutral collision frequency is much higher than that of the ion-neutral collision frequency the electrons are responsible in great part for the ionization.

If the Debye length is smaller than the Larmor radius of the ions but larger than that of the electrons it is plausible to imagine the electrons to carry almost all the current in the sheet with the ions moving in a quasi linear trajectory inside the sheet. As a result an electric field due to the charge separation occurs in the direction of the motion. It is clear that conservation of momentum will require the momentum flux of the ions to be balanced by the magnetic pressure of the ions.

In the model considered here a cold plasma is imagined to move toward a neutral gas and complete ionization is assumed to take place at the sheet edge. In a sense the ionization front acts as a source of ions and electrons.

The paper is divided in two parts. In the first a simplified theory for the structure of the sheet is discussed. In the second some experimental observations are examined in the light of the theory of part 1.

THE STRUCTURE OF THE CURRENT SHEET

A two fluid theory is developed for the determination of the details of the sheet. In accordance with the discussion of the previous section the sheet is assumed to move at a constant velocity v_s. It is further stipulated that the electrons Larmor radii are much smaller than the interelectrode distances and of the current sheet thickness as well.

If the cyclotron frequency of the electrons is much higher than the collision frequency (electron-neutral collisions) the treatment can follow the assumption of a collision free plasma. On the other hand the ions Larmor radius is considered to be much smaller than the characteristic dimension of the apparatus but can be larger than the thickness of the current sheet. Under these conditions it is plausible to assume the electrons to carry a major portion of the current. Indeed the ratio of the transverse velocities (in the z-direction) of the ions and electrons is actually equal to the ratio of their masses. It will then be shown that the magnetic forces can be neglected in as far as the ions are concerned.

The small size of the electron Larmor radius will allow one to use a hydrodynamic approximation for the electron fluid in the treatment of this problem. The motions of the ions too, can be described by a fluid model because the absence of collisions and the negligible effect of the magnetic forces will cause the ions to move as a quasi one-dimensional fluid.

The velocities of the electrons can be approximately estimated by their drifts so that one finds at once

$$v_y = E_z/B_x \tag{1}$$

and

$$v_z = -E_y/B_x . \tag{2}$$

where v_y, v_z are the velocities of the electron fluid in the y and z direction, respectively, E_y and E_z are the electric fields intensities in the same direction and B_x is the resulting magnetic field.

On the other hand for the ions one obtains by means of the momentum equation for steady state and for the conditions where all quantities are functions of the y coordinate only

$$m_i \, v_i \frac{dv_i}{dy} = e(E_y + v_{i,z} \, B_x) \tag{3}$$

In the above v_i is the velocity of the ion fluid in the y direction referred to a frame fixed with respect to the current sheet, while $v_{i,z}$ is the velocity of the ion fluid in the z direction. Substituting from (2) one finds

$$m_i \frac{d(v_i^2/2)}{dy} = e \, E_y \, (1 - \frac{v_{i,z}}{v_z}) \tag{4}$$

But $\frac{v_{i,z}}{v_x} \ll 1$, so that it can be neglected and Eq. (4) indicates that the kinetic energy of the ions are found to be derivable from a potential produced by the charge separation such that

$$m_i \, (\frac{v_i^2}{2}) = (\frac{m_i \, v_s^2}{2} - e\phi) \tag{5}$$

Hence at the edge of the sheet where the field vanishes the velocity v_i tends to the value v_s.

Now it is well known[7,8] that whenever $(\underline{E} + \underline{v} \times \underline{B}) = 0$, then for steady state conditions a simple dependence exists between the density of the conducting fluid and the magnetic field whenever two dimensional flow conditions prevail. This is seen readily from

$$\nabla \times \underline{E} = -\nabla \times (\underline{v} \times \underline{B}) = 0 \tag{6}$$

the latter step following from Faraday's relation and by comparing the expanded term

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[7] G. S. Golitsyn, J.E.T.P. 34, 473 (1958).

[8] A similar treatment has been followed by C. Longmire in his "Notes on Plasma Physics." (Los Alamos, 1958) (unpublished).

$$\nabla \times (\underline{v} \times \underline{B}) = (\underline{v} \cdot \nabla)\underline{B} + \underline{B}(\nabla \cdot v) = 0 \tag{7}$$

with the relation for the conservation of electrons

$$\nabla \cdot (n_e \underline{v}) = 0 = (\underline{v} \cdot \nabla)n_e + n_e(\nabla \cdot \underline{v}) \tag{8}$$

In the above equation (7) the solenoidal property of $\underline{B}$ has been used as well as the fact that $\underline{v}$ does not vary along the direction of $\underline{B}$.

Consequently it follows that here,

$$\frac{n_e}{B_x} = \text{constant} = \frac{n_o}{B_o}\text{, say.} \tag{9}$$

There are some simple relations between the density of the electrons and ions and the potential ϕ. Indeed manipulation of Amperes relation for the current

$$e\, n_e\, v_z = \frac{1}{\mu_o}\,\frac{dB_x}{dy} \tag{10}$$

in which the premise is made that the electrons carry all the current, together with (2) and (9) leads to

$$n_e = \frac{\mu_o e n_o^{\,2}}{B_o^{\,2}}\,(\phi + \phi_o) \tag{11}$$

where ϕ_o is a constant such that the density n_e tends to n_o whenever ϕ vanishes, i.e.

$$\phi_o = \frac{B_o^{\,2}}{\mu_o e n_o}\,. \tag{12}$$

Since all quantities are functions of y only, the conservation of charges yields

$$n_e\, v_y = n_i\, v_i = n_o\, v_s\,. \tag{13}$$

The two particle fluxes are indeed equal since an equal flux of particles of either charges penetrates the edge of the current sheet.

Combination of (11), (13) and (5) leads to an expression for Poisson's equation in which the potential is defined by the differential equation

$$\frac{d^2\phi}{dy^2} = -\frac{e}{\varepsilon_o}(n_i - n_e)$$

$$= -\frac{e}{\varepsilon_o}\left(\frac{n_o}{\sqrt{1 - \frac{2e}{m_i v_s^2}\phi}} - n_o\left(\frac{\mu_o n_o e}{B_o^2}\phi + 1\right)\right). \qquad (14)$$

Introducing the transformations

$$\psi = \left(\frac{2e}{m_i v_s^2}\right)\phi$$

$$y = \left(\frac{2e^2 n_o}{\varepsilon_o m_i v_s^2}\right)^{1/2} \zeta$$

and the parameter

$$\alpha = \frac{\mu_o n_o m_i v_s^2}{2B_o^2},$$

The above equation can be made dimensionless. Actually it becomes

$$\frac{d^2\psi}{d\zeta^2} = \left(\alpha\psi + 1 - \frac{1}{\sqrt{1 - \psi}}\right). \qquad (15)$$

The solution for this equation for a few values of the parameter α and for the boundary condition of a vanishing electric field and the potential is shown in Fig. (2).

For $\alpha < 1/2$, the solutions approaches asymptotically the functional behavior $\psi \sim e^{-\sqrt{1/2 - \alpha}\,\zeta}$. When the parameter α is greater than $\frac{1}{2}$ the potential ψ oscillates. For very large α the period is approximately proportional to $\frac{1}{\sqrt{\alpha - 1/2}}$.

The magnetic field obtained from the above relation is given in dimensionless form as

$$\frac{B_x}{B_o} = (\alpha\psi + 1) \tag{16}$$

and its behavior is shown in Fig. (3).

The actual thickness ℓ of the current sheet is determined from the condition

$$\begin{aligned} I &= -e \int_0^{\ell} n_e \, v_z \, dy \\ &= +e \int_0^{\ell} \frac{n_e}{B_x} E_y \, dy \end{aligned} \tag{17}$$

By virtue of (9), however, one can simplify the above to be

$$\begin{aligned} I &= -e \frac{n_o}{B_o} \int_0^{\ell} \frac{d\phi}{dy} \, dy \\ &= -e \frac{n_o}{B_o} \phi(\ell) \end{aligned} \tag{18}$$

In view of the periodic character of ϕ for $\alpha > \frac{1}{2}$ the current sheet may be several periods thick. This is not true however for $\alpha < 1/2$ in which case the thickness as defined by (18) becomes finite.

EXPERIMENTAL PROCEDURE

The general layout is shown schematically in Fig. 4. The discharge of the condenser bank takes place between two concentric electrodes. The current sheet which forms is fairly planar.

The magnetic field between the electrodes is detected by small probes similar in construction to the design of Karr[9]. Two probes are used for the mapping of the field. By placing the probes in the same plane the azimuthal dependence can be checked. Axial positioning of the probes on the other hand allows the determination of the structure of the sheet at different stations along the axis.

The important operating parameters are summarized in the table given below.

Maximum Condenser voltage	up to 12Kv
Condenser bank capacitance	30μF
Polarity: center electrode	negative
Electrode length	24 cms
Radius of inner electrode	3.5 cm
Radius of outer electrode	4.75 cm
Time to current maximum	1.75 μsec
Gas pressure: varied between	0.1-2mm
Gases used	H_2 and A

In order to verify the theory discussed in the previous section a series of measurements were done, to obtain the intensity of the magnetic field behind the sheet as well as the frequency of the oscillations in the plasma. Typical oscillograms for two different gases and different conditions are shown in Fig. 5.

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9 H. J. Carr, "The Plasma in a Magnetic Field." (Stanford University Press, 1958), p. 40 ff.

For the range of experimental values of the pressure used (0.1 - 2mm) the frequencies varied between 0.6 and 1.5 megacycles. By adjusting the voltage on the capacitor bank to different values it was possible to change the magnetic field and hence the Alfvén velocity. Magnetic field intensities between one and 6 kilogauss were thus obtained in this fashion.

For each experiment the velocity of the sheet was calculated from a determination of the time of flight of the sheet between the two probes. The result of all these measurements has been condensed in graphical form in Fig. 6 which shows the dependence of the frequency on the coefficient α.

It appears that the mean features of the theory are verified. Indeed no oscillations were noticed when the factor α was too low.

An important conclusion arrived here is that the assumption of the ions carrying a small fraction of the current seems to be a fairly good one. It thus implies that the ions gain energy indirectly from transfer through electron-ions collisions. This fact may be of importance in clarifying the onset of a shock in an electrically driven shock tube.

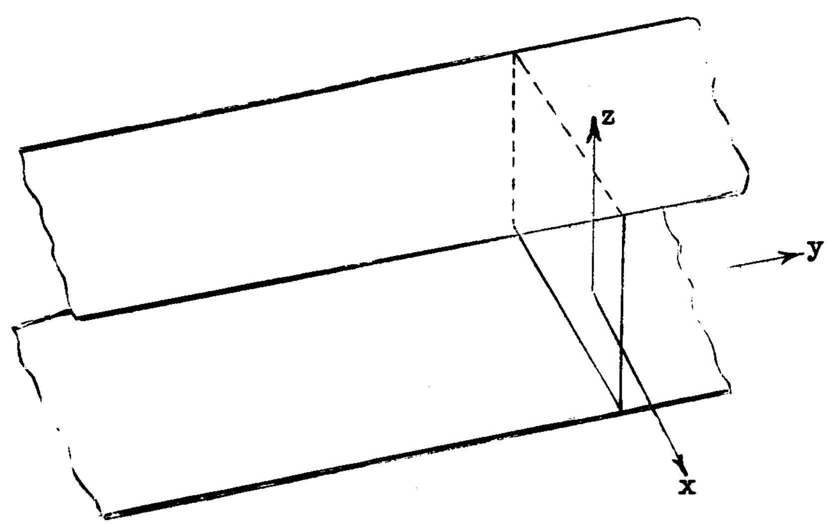


Fig. 1: Electrode configuration and coordinate axis used in the text.

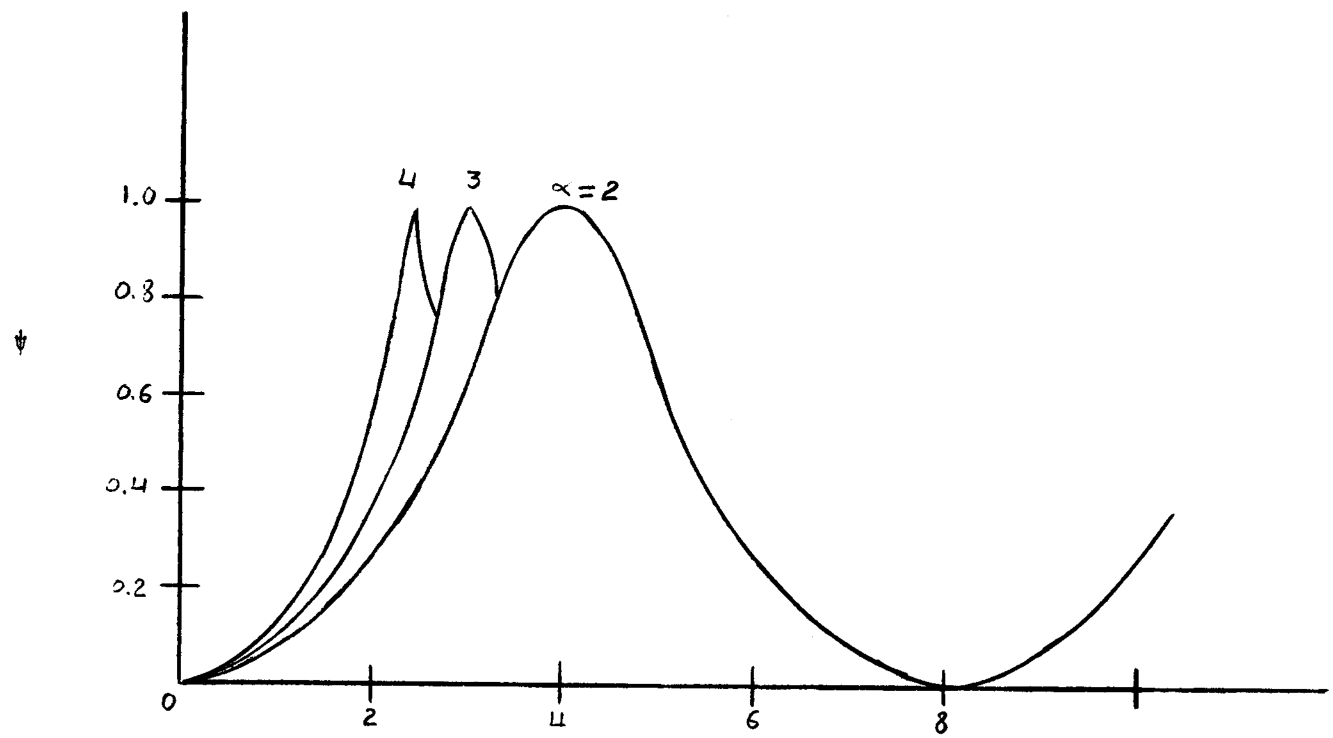


Fig. 2: Dependence of the potential on the dimensionless distance ζ.

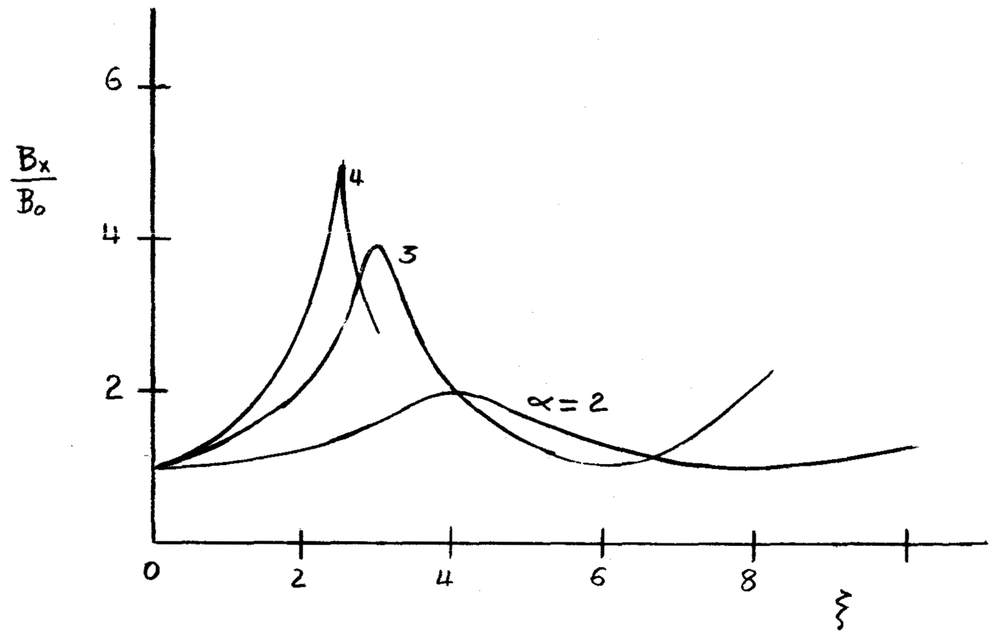


Fig. 3: Functional behavior of the magnetic field referred to its value at the sheet edge with respect to ζ.

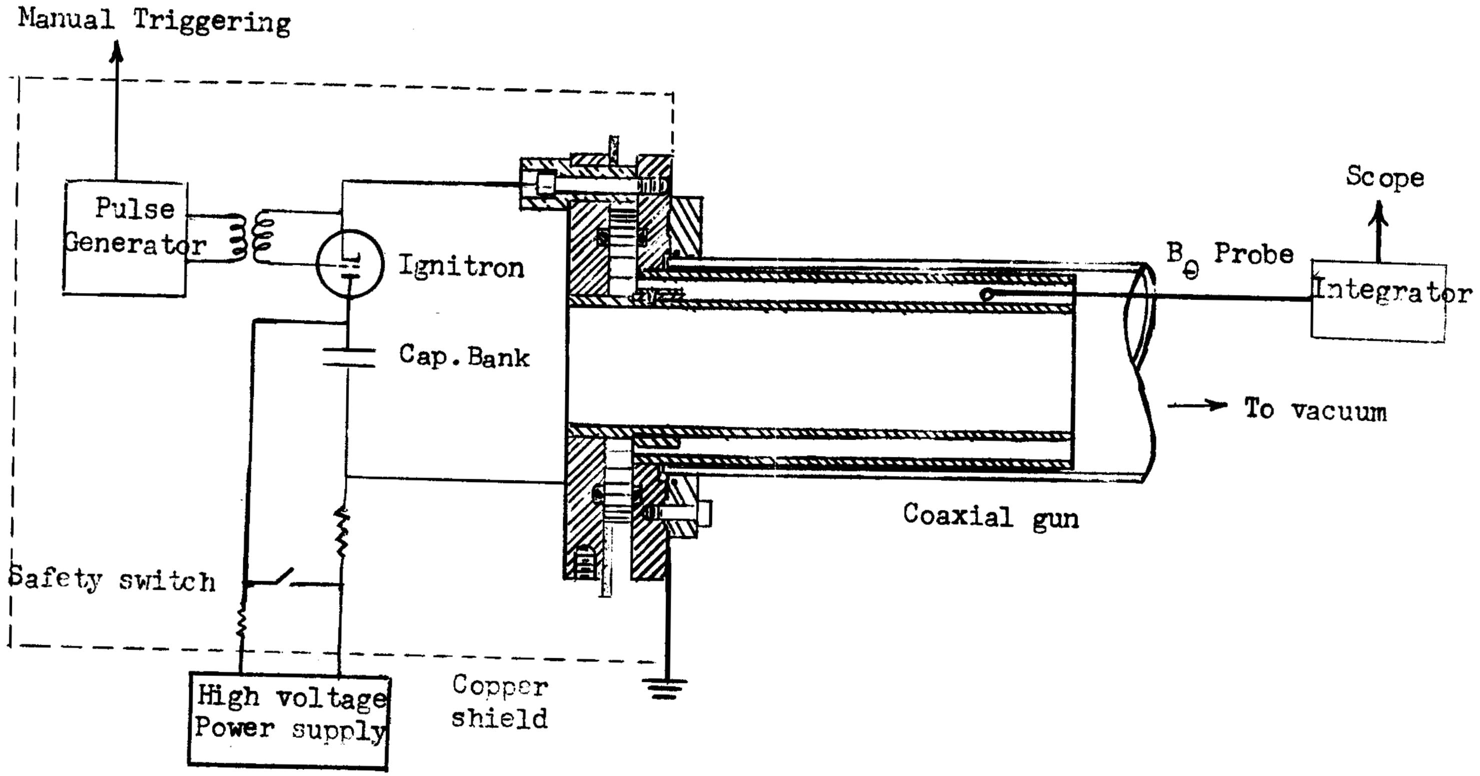
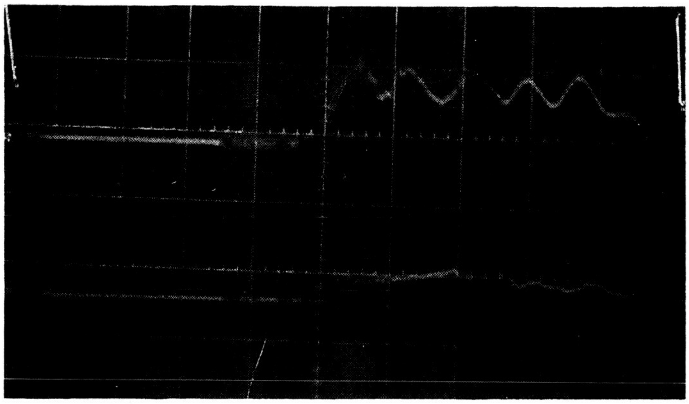


Fig. 4: Schematic Diagram for the Apparatus Used.



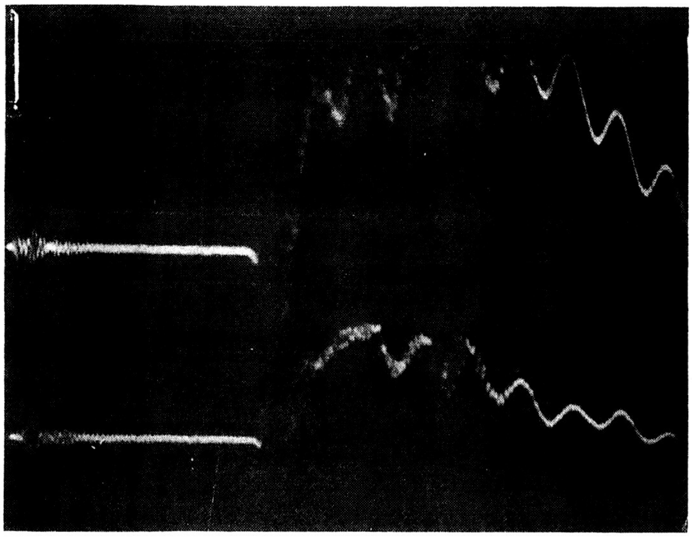
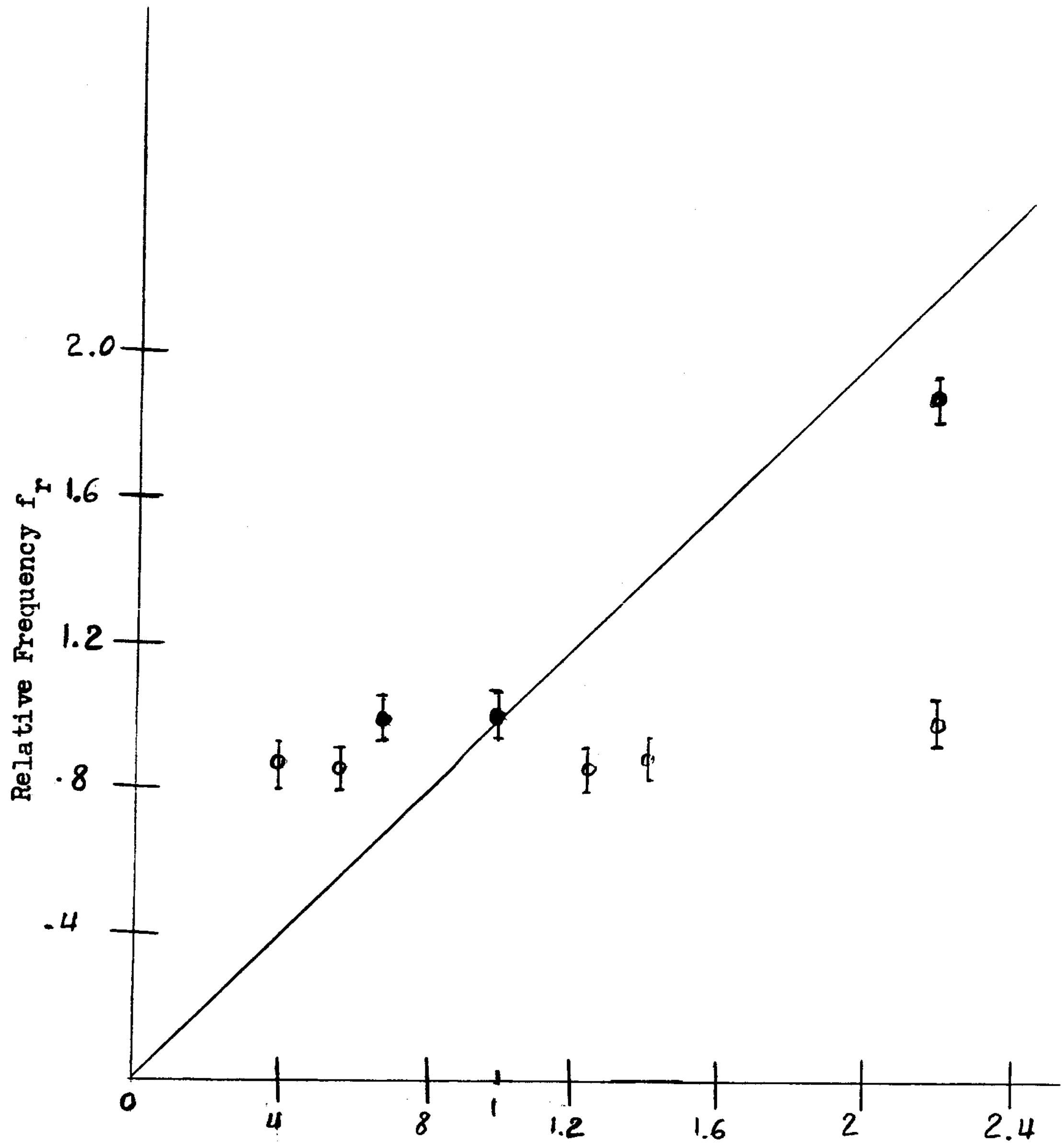


Fig. 5: Two oscillograms for the spatial behavior of the magnetic field. (a) refers to Hydrogen at .2 mm Hg (b) refers to Argon at .2 mm Hg. The curves denoted by (1) and (2) are for two probes separated by an axial distance of about 9 cms. The relative phasing of the signal picked up by (1) and (2) denotes the true travelling wave behavior of the sheet structure.



Relative value of parameter α

Fig. 6: Dependence of the relative magnitude of the periods on the relative values of the parameter. For a perfect check of the theory the points should fall on the 45° (solid) line. The full circles refer to Hydrogen, the open circles to Argon. As the sheet velocity is not strongly dependent on the pressure, α is almost inversely proportional to the Alfvén velocity.